The one-way CNOT simulation

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In this paper we present the complete simulation of the quantum logic CNOT gate in the one-way model, that consists entirely of one-qubit measurements on a particular class of entangled states.

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I. INTRODUCTION

The one-way quantum computation model provides a new and alternative way to build quantum computer, more efficiently than the logic network model [1],[2]. The quantum gates simulation is operated by highly entangled cluster states of a large number of qubits. The entangled state of the cluster serves as a universal "substratum" for any quantum gate. To implement unitary transformation it suffices to make only one-qubit measurements. The measurement type and the order in which the measurements are performed, determine the implemented quantum algorithm. The calculations results are written onto the "output" qubits of the cluster, which are thereby the quantum register. The output qubits are read with one-qubit measurements too, by which the classical readout is obtained. The result of previous measurements determine in which basis the output qubits need to be measured for the final readout, or, if the readout measurements are in the $\sigma_x, \sigma_y, \sigma_z$ eigenbasis, how the readout measurements have to be interpreted. The individual measurement results are random but correlated. These correlations enable the quantum computation.

The entangled cluster states can be created efficiently in any quantum system with a quantum Ising-type interaction between two-state particles in a lattice configuration at very low temperatures. Experimentally, the cluster states can be created as follows. First, a product state:

$$| + >_C = \bigotimes_{a\in C} | + >_a \tag{1}$$

where "a" is the qubit number of the cluster, is prepared putting all qubits in the $| + >$ state.

Second, the unitary transformation $S^{(C)}$:

$$S^{(C)} = \prod_{a,b\in C|b-a\in\gamma_f} S^{ab} \tag{2}$$

is applied to the state $| + >_C$. The letter "f" shows the cluster dimension. For the cases of dimension $f = 1, 2, 3$ we have:

$$\begin{aligned}\gamma_1 &= \{1\}\\ \gamma_2 &= \{(1,0)^T, (0,1)^T\}\\ \gamma_3 &= \{(1,0,0)^T, (0,1,0)^T, (0,0,1)^T\}.\end{aligned} \tag{3}$$

The two-qubit transformation $S^{(ab)}$ is such that the state $| 1 >_a \otimes | 1 >_b$ acquires a phase of π under its action whereas the remaining states

$$| 0 >_a \otimes | 0 >_b, \quad | 0 >_a \otimes | 1 >_b, \quad | 1 >_a \otimes | 0 >_b$$

acquire no phase. Thus, $S^{(ab)}$ has the form:

$$\begin{aligned}S^{(ab)} &= \frac{1}{2}(\mathbf{1}^{(a)} \otimes \mathbf{1}^{(b)} + {\sigma^{(a)}}_z \otimes \mathbf{1}^{(b)}\\ &+ \mathbf{1}^{(a)} \otimes {\sigma^{(b)}}_z - {\sigma^{(a)}}_z \otimes {\sigma^{(b)}}_z).\end{aligned} \tag{4}$$

The unitary transformation $S^{(C)}$ thereby acts only onto the near neighboring qubits .

The cluster state $| + >_C$ obeys the eigenvalue equations:

$$| \phi >_C = S{\sigma^{(a)}}_x S^\dagger | \phi >_C, \quad \forall a \in C. \tag{5}$$

where, for brevity, $S = S^C$.

To obtain $S{\sigma^{(a)}}_x S^\dagger$, one observes that:

$$\begin{aligned}S^{(ab)}{\sigma^{(a)}}_x {S^{(ab)}}^\dagger | \phi >_C &= {\sigma^{(a)}}_x \otimes {\sigma^{(b)}}_z | \phi >_C\\ S^{(ab)}{\sigma^{(a)}}_x {S^{(ab)}}^\dagger | \phi >_C &= {\sigma^{(a)}}_z \otimes {\sigma^{(b)}}_x | \phi >_C\end{aligned} \tag{6}$$

and

$$S^{(ab)}{\sigma^{(c)}}_x {S^{ab}}^\dagger | \phi >_C = {\sigma^{(c)}}_x | \phi >_C, \quad \forall c \in C\backslash\{a, b\}. \tag{7}$$

Furthermore for the Pauli phase flip operators σ_z:

$$S^{(ab)}{\sigma^{(d)}}_z {S^{(ab)}}^\dagger = {\sigma^{(d)}}_z, \quad \forall d \in C\backslash\{a, b\}. \tag{8}$$

From the preceding equations, one gets:

$$S\sigma^{(a)}{}_x S^\dagger = \sigma^{(a)}{}_x \bigotimes_{b\in nbgh(a)} \sigma^{(b)}{}_z. \tag{9}$$

Thus the cluster states $\mid \phi >_C$ generated from $\mid + >_C$ via the transformation $S^{(C)}$ obeys the eigenvalue equations:

$$K^a \mid \phi_{(\kappa)} >_C = \mid \phi_{(\kappa)} >_C \tag{10}$$

with

$$K^a = \sigma_x{}^a \bigotimes_{b\in nbgh(a)} \sigma_z{}^b. \tag{11}$$

The CNOT gate is a controlled gate with two input named control and target respectively. The cluster structure which is needed for the CNOT gate is showed in fig.1. The cluster is composed of 15 qubits. The qubits 1 e 9 are the input qubits of the gate, the qubits 7 e 15 are the output qubits. The gate simulation is obtained in two step:

1. entangled state formation (section III);
2. measurements onto the cluster (section IV).

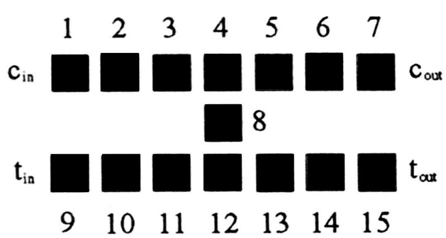

FIG. 1: cluster's structure for CNOT gate.

II. TABULAR NOTATION

To simplify the presentation of calculations we developed and introduced a new notation, which we named "tabular notation".

Any cluster state is a sum of many tensorial products, in which are written the state of all cluster qubits. We can show them as items of a row in a table which has a number of columns equal to qubits number plus one. The first column is named "sign", the remaining ones give the qubit position in the cluster. Into the column "sign" the algebraic sign of the tensorial products is written and into the numbered columns the qubits' states. Any tensorial product is thereby written into a line, and the cluster state is represented by the whole table. With this notation, we can calculate the cluster state more easily without mistakes.

III. ENTANGLED STATE FORMATION

Refering to fig.1, the cluster for the CNOT gate is divided in subclusters formed by 1, 2, 3, or 4 qubits. The entanglement operator is applied to all subclusters. Then, the subclusters are "repaired" with the interaction of marginal qubits. In this way, we can piece together the whole cluster. We use the following subclusters:

1. from qubit 1 to qubit 3;
2. from qubit 4 to qubit 7;
3. from quibt 9 to qubit 11;
4. from qubit 12 to qubit 15;
5. the qubit 8.

We show now the way the operator $S^{(C)}$ is applied. Initially, we take two qubits, in the $\mid + >$ state, that is:

$$\mid \Psi >_2 \ = \ \mid + >_1 \mid + >_2 \tag{12}$$

and then apply to this state the operator $S^{(2)}$. So, the entangled state:

$$S^{(2)} \mid \Psi >_2 \ = \ \mid 0 >_1 \mid + >_2 + \mid 1 >_1 \mid - >_2 . \tag{13}$$

is obtained.

After, we take three qubits in the $\mid + >$:

$$\mid \Psi_3 > \ = \ \mid + >_1 \mid + >_2 \mid + >_3, \tag{14}$$

appling the operator $S^{(3)}$, one obtains:

$$\begin{aligned} S^{(3)} \mid \Psi >_3 \ &= \ \mid + >_1 \mid 0 >_2 \mid + >_3 \\ &+ \ \mid - >_1 \mid 1 >_2 \mid - >_3 . \end{aligned} \tag{15}$$

In the case of 4 qubits:

$$\mid \Psi >_4 \ = \ \mid + >_1 \mid + >_2 \mid + >_3 \mid + >_4 \tag{16}$$

the result is:

$$\begin{aligned} S^{(4)} \mid \Psi >_4 \ &= \ \mid 0 >_1 \mid + >_2 \mid 0 >_3 \mid + >_4 \\ &+ \ \mid 0 >_1 \mid - >_2 \mid 1 >_3 \mid - >_4 \\ &+ \ \mid 1 >_1 \mid - >_2 \mid 0 >_3 \mid + >_4 \\ &+ \ \mid 1 >_1 \mid + >_2 \mid 1 >_3 \mid - >_4 . \end{aligned} \tag{17}$$

Whereas in the case of 5 qubits:

$$\mid \Psi >_{1,2,3,4,5} = \mid \psi_{in} >_1 \mid + >_2 \mid + >_3 \mid + >_4 \mid + >_5 \tag{18}$$

the result is:

$$
\begin{aligned}
S^{(5)} \mid \Psi >_{1,2,3,4,5} &= \mid \psi_{in} >_1 \mid 0 >_2 \mid + >_3 \mid 0 >_4 \mid + >_5 \\
&+ \mid \psi_{in} >_1 \mid 0 >_2 \mid - >_3 \mid 1 >_4 \mid - >_5 \\
&+ \mid \psi_{in}* >_1 \mid 1 >_2 \mid - >_3 \mid 0 >_4 \mid + >_5 \\
&+ \mid \psi_{in}* >_1 \mid 1 >_2 \mid + >_3 \mid 1 >_4 \mid - >_5
\end{aligned} \tag{19}
$$

where the state $\mid \psi_{in} >$ and $\mid \psi_{in}* >$ are linear combinations of the states $\mid 0 >$ and $\mid 1 >$:

$$\mid \psi_{in} > \; = \; a \mid 0 > +b \mid 1 > \tag{20}$$

$$\mid \psi_{in}* > \; = \; a \mid 0 > -b \mid 1 > . \tag{21}$$

The states obtained by us are different from the results published by Raussendorf and Briegel [2], [3]. Their result for the cluster of 4 and 5 qubits are respectivily:

$$
\begin{aligned}
S^{(4)} \mid \Psi >_4 &= \mid + >_1 \mid 0 >_2 \mid + >_3 \mid 0 >_4 \\
&+ \mid + >_1 \mid 0 >_2 \mid - >_3 \mid 1 >_4 \\
&+ \mid - >_1 \mid 1 >_2 \mid - >_3 \mid 0 >_4 \\
&+ \mid - >_1 \mid 1 >_2 \mid + >_3 \mid 1 >_4 .
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
S^{(5)} \mid \Psi >_{1,2,3,4,5} &= \mid \psi_{in} >_1 \mid 0 >_2 \mid - >_3 \mid 0 >_4 \mid - >_5 \\
&- \mid \psi_{in} >_1 \mid 0 >_2 \mid + >_3 \mid 1 >_4 \mid + >_5 \\
&- \mid \psi_{in}* >_1 \mid 1 >_2 \mid + >_3 \mid 0 >_4 \mid - >_5 \\
&+ \mid \psi_{in}* >_1 \mid 1 >_2 \mid - >_3 \mid 1 >_4 \mid + >_5 .
\end{aligned} \tag{23}
$$

It is important to note that their results are not compatible between them.

We used our results to calculate the cluster state for 15 qubits. The relevant results are reported in appendix B. In this way, we have obtained the expression of the cluster state for the one-way CNOT gate.

IV. MEASUREMENTS ONTO THE CLUSTER

We name the input qubits $C_I(g)$, the output qubits $C_O(g)$, and the measurements cluster $C_M(g)$. The implementation of unitary transformation $C(CNOT)$ is strictly tied to the fundamental theorem of the one-way quantum computation (see appendix B).

We applied the fundamental theorem for the analysis of CNOT gate. The qubits 1 and 8 give $C_I(g)$, the qubits 7 e 15 $C_O(g)$, all others qubits $C_M(g)$. Let $\mid \phi >$ be a cluster state on $C(CNOT)$ which satisfies the set of eigenvalue equations (10). From these basic eigenvalue equations one gets:

$$
\begin{aligned}
\mid \phi > &= K^{(1)}K^{(3)}K^{(4)}K^{(5)}K^{(7)}K^{(8)}K^{(13)}K^{(15)} \mid \phi > \\
&= -{\sigma_x}^{(1)}{\sigma_y}^{(3)}{\sigma_y}^{(4)}{\sigma_y}^{(5)}{\sigma_x}^{(7)}{\sigma_y}^{(8)}{\sigma_x}^{(13)}{\sigma_x}^{(15)} \mid \phi > &(24)\\
\mid \phi > &= K^{(2)}K^{(3)}K^{(5)}K^{(6)} \mid \phi > \\
&= {\sigma_z}^{(1)}{\sigma_y}^{(2)}{\sigma_y}^{(3)}{\sigma_y}^{(5)}{\sigma_y}^{(6)}{\sigma_z}^{(7)} &(25)\\
\mid \phi > &= K^{(9)}K^{(11)}K^{(13)}K^{(15)} \mid \phi > \\
&= {\sigma_x}^{(9)}{\sigma_x}^{(11)}{\sigma_x}^{(13)}{\sigma_x}^{(15)} \mid \phi > &(26)\\
\mid \phi > &= K^{(5)}K^{(6)}K^{(8)}K^{(10)}K^{(12)}K^{(14)} \mid \phi > \\
&= {\sigma_y}^{(5)}{\sigma_y}^{(6)}{\sigma_z}^{(7)}{\sigma_y}^{(8)}{\sigma_z}^{(9)}{\sigma_x}^{(10)}{\sigma_y}^{(12)}{\sigma_x}^{(14)}{\sigma_z}^{15)} \mid \phi > . &(27)
\end{aligned}
$$

If the qubits 10, 11, 13 and 14 are measured in the σ_x eigenbasis and the qubits 2, 3, 4, 5, 6, 8 and 12 are measured in the σ_y eigenbasis, then the measurement results $s_2, s_3, s_4, s_5, s_6, s_8, s_{10}, s_{11}, s_{12}, s_{13}, s_{14} \in \{0, 1\}$ are obtained. The cluster state eigenvalue equations (28), (29), (30), (31) lead to the following eigenvalue equations for the projected state:

$$
\begin{aligned}
&{\sigma_x}^{(1)}{\sigma_x}^{(7)}{\sigma_x}^{(15)} \mid \psi > \\
&= (-1)^{1+s_3+s_4+s_5+s_8+s_{13}} \mid \psi > &(28)\\
&{\sigma_z}^{(1)}{\sigma_z}^{(7)} \mid \psi > \\
&= (-1)^{s_2+s_3+s_5+s_6} \mid \psi > &(29)\\
&{\sigma_x}^{(9)}{\sigma_x}^{(15)} \mid \psi > \\
&= (-1)^{s_{11}+s_{13}} \mid \psi > &(30)\\
&{\sigma_z}^{(7)}{\sigma_z}^{(9)}{\sigma_z}^{(15)} \mid \psi > \\
&= (-1)^{s_5+s_6+s_8+s_{10}+s_{12}+s_{14}} \mid \psi > . &(31)
\end{aligned}
$$

Therein, qubits 1 and 7 represent the input and output for the control qubit and qubits 9 and 15 represent the input and output for the target qubit. Writing the CNOT unitary operation on control and target qubits $CNOT(c,t)$, we find:

$$
\begin{aligned}
CNOT(c,t){\sigma_x}^{(c)}CNOT(c,t) &= {\sigma_x}^{(c)}{\sigma_x}^{(t)} && (32)\\
CNOT(c,t){\sigma_z}^{(c)}CNOT(c,t) &= {\sigma_z}^{(c)} && (33)\\
CNOT(c,t){\sigma_x}^{(t)}CNOT(c,t) &= {\sigma_x}^{(t)} && (34)\\
CNOT(c,t){\sigma_z}^{(t)}CNOT(c,t) &= {\sigma_z}^{(c)}{\sigma_z}^{(t)}. && (35)
\end{aligned}
$$

Comparing these equations to the eigenvalue equations (32) to (35), one clearly sees that measurements do indeed realize a CNOT gate. Furthermore, after reading off the operator $U_{\sum}$ using equations (A2) and (A4) and propagating the byproduct operators through to the output side of the CNOT gate, one finds the following expression for the byproduct operators of CNOT gate:

$$U_{\sum,CNOT} = {\sigma_x}^{(c)^{\gamma_x{}^{(c)}}} {\sigma_x}^{(t)^{\gamma_x{}^{(t)}}} {\sigma_z}^{(c)^{\gamma_z{}^{(c)}}} {\sigma_z}^{(t)^{\gamma_z{}^{(t)}}} \quad (36)$$

with

$$
\begin{aligned}
{\gamma_x}^{(c)} &= s_2+s_3+s_5+s_6\\
{\gamma_x}^{(t)} &= s_2+s_3+s_8+s_{10}+s_{12}+s_{14}\\
{\gamma_z}^{(c)} &= s_1+s_3+s_4+s_5+s_8+s_9+s_{11}+1\\
{\gamma_z}^{(t)} &= s_9+s_{11}+s_{13}. \qquad (37)
\end{aligned}
$$

The measurements set applied to the cluster is showed in fig.2.

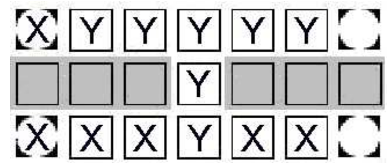

FIG. 2: measurements onto CNOT cluster.

V. CONCLUSIONS

In this note we presented theoretical results on the simulation of CNOT gate, based on the one-way quantum computation model. The first complete simulation of CNOT gate onto a cluster state is exhibited. Being the CNOT a universal gate, its simulation makes valid the one-way model of quantum computer.

[1] M. A. Nielsen, I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press.

[2] R. Raussendorf, H. J. Briegel, *A One-Way Quantum Computer*, Physical Review Letters, vol. 86, No. 22, pag. 5188-5191, 2001.

[3] R. Raussendorf, D. E. Browne, H. J. Briegel, *Measurement-based quantum computation on cluster stastes*, arXiv: quantum-ph/0301052 v2, 2005.

APPENDIX A: THEOREM

The fundamental theorem of the one-way quantum computation is the following [3].

Theor.:
be
$C(g) = C_I(g) \cup C_M(g) \cup C_O(g)$
with
$C_I(g) \cap C_M(g) = C_I(g) \cap C_O(g) = C_M(g) \cap C_O(g) = \emptyset$
a cluster for the simulation of a gate "g", realizing the unitary trasformation U *and* $\mid \phi >_{C(g)}$ *the cluster state on the cluster* $C(g)$*. Ones measures the qubits* $C_M(g)$*, that is projects the initial state* $\mid \phi >_{C(g)}$ *in the new state* $\mid \psi >_{C(g)} = {P_{\{s\}}}^{(C_M(g))} \mid \phi >_{C(g)}$*. The measuremnt pattern* $\mathcal{M}^{(\mathcal{C})}$ *which is applied to the* $C_M(g)$ *qubits is a set of vectors:*

$$\mathcal{M}^{(\mathcal{C})} = \{\vec{r}_a \in S^2 \mid a \in \mathcal{C}\}. \quad (A1)$$

This vectors determine the basis for all measurements onto the cluster.
Suppose, the new state $\mid \psi >_{C(g)}$ *obeys the 2n eigenvalue equations:*

$$
\begin{aligned}
{\sigma_x}^{(C_I(g),i)}(U{\sigma_x}^{(i)}U^\dagger)^{(C_O(g))} \mid \psi >_{C(g)} &= && (A2)\\
(-1)^{\lambda_{x,i}} \mid \psi >_{C(g)} & &&\\
{\sigma_z}^{(C_I(g),i)}(U{\sigma_z}^{(i)}U^\dagger)^{(C_O(g))} \mid \psi >_{C(g)} &= && (A3)\\
(-1)^{\lambda_{z,i}} \mid \psi >_{C(g)} & && (A4)
\end{aligned}
$$

where $\lambda_{x,i}, \lambda_{z,i} \in \{0,1\}$ *e* $1 \le i \le n$ *and "n" shows the number of logic qubits which must be processed. Then, the logic gate "g" which acts onto the initial state* $\mid \psi_{in} >$ *is carried out onto the cluster* $C(g)$ *appling to the qubits* $C_M(g)$ *the projector's operator* ${P_{\{s\}}}^{(C_M(g))}$ *and to the qubits* $C_I(g)$ *the operator* σ_x*.*

In this way, the input and the output state in the simulation of "g" are related via:

$$| \psi_{out} >= UU_{\sum} \mid \psi_{in} > \tag{A5}$$

where $U_{\sum}$ is a byproduct operator given by:

$$U_{\sum} = \bigotimes_{i=1\ldots n} (\sigma_z{}^{[i]})^{s_i+\lambda_{x,i}} (\sigma_x{}^{[i]})^{\lambda_{z,i}}. \tag{A6}$$

APPENDIX B: RELEVANT RESULTS

The entangled state $S^{(3)} \mid \Psi_{1,2,3} >$ is represented by:

sign	1	2	3
+	ψ_{in}	0	+
+	$\psi_{in}*$	1	-

The entangled state $S^{(3)} \mid \Psi_{9,10,,11} >$ is represented by:

sign	9	10	11
+	ψ_{in}	0	+
+	$\psi_{in}*$	1	-

The entangled state $S^{(4)} \mid \Psi_{4,5,6,7} >$ is represented by:

sign	4	5	6	7
+	0	+	0	+
+	0	−	1	−
+	1	−	0	+
+	1	+	1	−

The entangled state $S^{(4)} \mid \Psi_{12,13,14,15} >$ is represented by:

sign	12	13	14	15
+	0	+	0	+
+	0	−	1	−
+	1	−	0	+
+	1	+	1	−

The entangled state $S^{(3\cdot 4)} \mid \Psi_{1,2,3,4,5,6,7} >$ is represented by:

sign	1	2	3	4	5	6	7
+	ψ_{in}	0	+	0	+	0	+
+	ψ_{in}	0	+	0	-	1	-
+	ψ_{in}	0	-	1	-	0	+
+	ψ_{in}	0	-	1	+	1	-
+	$\psi_{in}*$	1	-	0	+	0	+
+	$\psi_{in}*$	1	-	0	-	1	-
+	$\psi_{in}*$	1	+	1	-	0	+
+	$\psi_{in}*$	1	+	1	+	1	-

The entangled state $S^{(11\cdot 12)} \mid \Psi_{9,10,11,12,13,14,15} >$ is represented by:

sign	9	10	11	12	13	14	15
+	ψ_{in}	0	+	0	+	0	+
+	ψ_{in}	0	+	0	-	1	-
+	ψ_{in}	0	-	1	-	0	+
+	ψ_{in}	0	-	1	+	1	-
+	$\psi_{in}*$	1	-	0	+	0	+
+	$\psi_{in}*$	1	-	0	-	1	-
+	$\psi_{in}*$	1	+	1	-	0	+
+	$\psi_{in}*$	1	+	1	+	1	-

For the state $S^{(4\cdot 8)} \mid \Psi_{1,2,3,4,5,6,7,8} >$ the result is:

sign	1	2	3	4	5	6	7	8
+	ψ_{in}	0	+	0	+	0	+	0
+	ψ_{in}	0	+	0	+	0	+	1
+	ψ_{in}	0	+	0	-	1	-	0
+	ψ_{in}	0	+	0	-	1	-	1
+	ψ_{in}	0	-	1	-	0	+	0
-	ψ_{in}	0	-	1	-	0	+	1
+	ψ_{in}	0	-	1	+	1	-	0
-	ψ_{in}	0	-	1	+	1	-	1
+	$\psi_{in}*$	1	-	0	+	0	+	0
+	$\psi_{in}*$	1	-	0	+	0	+	1
+	$\psi_{in}*$	1	-	0	-	1	-	0
+	$\psi_{in}*$	1	-	0	-	1	-	1
+	$\psi_{in}*$	1	+	1	-	0	+	0
-	$\psi_{in}*$	1	+	1	-	0	+	1
+	$\psi_{in}*$	1	+	1	+	1	-	0
-	$\psi_{in}*$	1	+	1	+	1	-	1

The last result is the state $S^{(8\cdot 12)} \mid \Psi_{1,\ldots,15} >$.The result is broken in two parts for typographical reasons:

sign	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
+	ψ_{in}	0	+	0	+	0	+	0	ψ_{in}	0	+	0	+	0	+
+	ψ_{in}	0	+	0	+	0	+	0	ψ_{in}	0	+	0	-	1	-
+	ψ_{in}	0	+	0	+	0	+	0	ψ_{in}	0	-	1	-	0	+
+	ψ_{in}	0	+	0	+	0	+	0	ψ_{in}	0	-	1	+	1	-
+	ψ_{in}	0	+	0	+	0	+	0	$\psi_{in}*$	1	-	0	+	0	+
+	ψ_{in}	0	+	0	+	0	+	0	$\psi_{in}*$	1	-	0	-	1	-
+	ψ_{in}	0	+	0	+	0	+	0	$\psi_{in}*$	1	+	1	-	0	+
+	ψ_{in}	0	+	0	+	0	+	0	$\psi_{in}*$	1	+	1	+	1	-
+	ψ_{in}	0	+	0	+	0	+	1	ψ_{in}	0	+	0	+	0	+
+	ψ_{in}	0	+	0	+	0	+	1	ψ_{in}	0	+	0	-	1	-
-	ψ_{in}	0	+	0	+	0	+	1	ψ_{in}	0	-	1	-	0	+
-	ψ_{in}	0	+	0	+	0	+	1	ψ_{in}	0	-	1	+	1	-
+	ψ_{in}	0	+	0	+	0	+	1	$\psi_{in}*$	1	-	0	+	0	+
+	ψ_{in}	0	+	0	+	0	+	1	$\psi_{in}*$	1	-	0	-	1	-
-	ψ_{in}	0	+	0	+	0	+	1	$\psi_{in}*$	1	+	1	-	0	+
-	ψ_{in}	0	+	0	+	0	+	1	$\psi_{in}*$	1	+	1	+	1	-
+	ψ_{in}	0	+	0	-	1	-	0	ψ_{in}	0	+	0	+	0	+
+	ψ_{in}	0	+	0	-	1	-	0	ψ_{in}	0	+	0	-	1	-
+	ψ_{in}	0	+	0	-	1	-	0	ψ_{in}	0	-	1	-	0	+
+	ψ_{in}	0	+	0	-	1	-	0	ψ_{in}	0	-	1	+	1	-
+	ψ_{in}	0	+	0	-	1	-	0	$\psi_{in}*$	1	-	0	+	0	+
+	ψ_{in}	0	+	0	-	1	-	0	$\psi_{in}*$	1	-	0	-	1	-
+	ψ_{in}	0	+	0	-	1	-	0	$\psi_{in}*$	1	+	1	-	0	+
+	ψ_{in}	0	+	0	-	1	-	0	$\psi_{in}*$	1	+	1	+	1	-
+	ψ_{in}	0	+	0	-	1	-	1	ψ_{in}	0	+	0	+	0	+
+	ψ_{in}	0	+	0	-	1	-	1	ψ_{in}	0	+	0	-	1	-
-	ψ_{in}	0	+	0	-	1	-	1	ψ_{in}	0	-	1	-	0	+
-	ψ_{in}	0	+	0	-	1	-	1	ψ_{in}	0	-	1	+	1	-
+	ψ_{in}	0	+	0	-	1	-	1	$\psi_{in}*$	1	-	0	+	0	+
+	ψ_{in}	0	+	0	-	1	-	1	$\psi_{in}*$	1	-	0	-	1	-
-	ψ_{in}	0	+	0	-	1	-	1	$\psi_{in}*$	1	+	1	-	0	+
-	ψ_{in}	0	+	0	-	1	-	1	$\psi_{in}*$	1	+	1	+	1	-
+	ψ_{in}	0	-	1	-	0	+	0	ψ_{in}	0	+	0	+	0	+
+	ψ_{in}	0	-	1	-	0	+	0	ψ_{in}	0	+	0	-	1	-
+	ψ_{in}	0	-	1	-	0	+	0	ψ_{in}	0	-	1	-	0	+
+	ψ_{in}	0	-	1	-	0	+	0	ψ_{in}	0	-	1	+	1	-
+	ψ_{in}	0	-	1	-	0	+	0	$\psi_{in}*$	1	-	0	+	0	+
+	ψ_{in}	0	-	1	-	0	+	0	$\psi_{in}*$	1	-	0	-	1	-
+	ψ_{in}	0	-	1	-	0	+	0	$\psi_{in}*$	1	+	1	-	0	+
+	ψ_{in}	0	-	1	-	0	+	0	$\psi_{in}*$	1	+	1	+	1	-
-	ψ_{in}	0	-	1	-	0	+	1	ψ_{in}	0	+	0	+	0	+
-	ψ_{in}	0	-	1	-	0	+	1	ψ_{in}	0	+	0	-	1	-
+	ψ_{in}	0	-	1	-	0	+	1	ψ_{in}	0	-	1	-	0	+
+	ψ_{in}	0	-	1	-	0	+	1	ψ_{in}	0	-	1	+	1	-
-	ψ_{in}	0	-	1	-	0	+	1	$\psi_{in}*$	1	-	0	+	0	+
-	ψ_{in}	0	-	1	-	0	+	1	$\psi_{in}*$	1	-	0	-	1	-
+	ψ_{in}	0	-	1	-	0	+	1	$\psi_{in}*$	1	+	1	-	0	+
+	ψ_{in}	0	-	1	-	0	+	1	$\psi_{in}*$	1	+	1	+	1	-
+	ψ_{in}	0	-	1	+	1	-	0	ψ_{in}	0	+	0	+	0	+
+	ψ_{in}	0	-	1	+	1	-	0	ψ_{in}	0	+	0	-	1	-
+	ψ_{in}	0	-	1	+	1	-	0	ψ_{in}	0	-	1	-	0	+
+	ψ_{in}	0	-	1	+	1	-	0	ψ_{in}	0	-	1	+	1	-
+	ψ_{in}	0	-	1	+	1	-	0	$\psi_{in}*$	1	-	0	+	0	+
+	ψ_{in}	0	-	1	+	1	-	0	$\psi_{in}*$	1	-	0	-	1	-
+	ψ_{in}	0	-	1	+	1	-	0	$\psi_{in}*$	1	+	1	-	0	+
+	ψ_{in}	0	-	1	+	1	-	0	$\psi_{in}*$	1	+	1	+	1	-
-	ψ_{in}	0	-	1	+	1	-	1	ψ_{in}	0	+	0	+	0	+
-	ψ_{in}	0	-	1	+	1	-	1	ψ_{in}	0	+	0	-	1	-
+	ψ_{in}	0	-	1	+	1	-	1	ψ_{in}	0	-	1	-	0	+
+	ψ_{in}	0	-	1	+	1	-	1	ψ_{in}	0	-	1	+	1	-
-	ψ_{in}	0	-	1	+	1	-	1	$\psi_{in}*$	1	-	0	+	0	+
-	ψ_{in}	0	-	1	+	1	-	1	$\psi_{in}*$	1	-	0	-	1	-
+	ψ_{in}	0	-	1	+	1	-	1	$\psi_{in}*$	1	+	1	-	0	+
+	ψ_{in}	0	-	1	+	1	-	1	$\psi_{in}*$	1	+	1	+	1	-

+	$\psi_{in}*$	1	-	0	+	0	+	0	ψ_{in}	0	+	0	+	0	+
+	$\psi_{in}*$	1	-	0	+	0	+	0	ψ_{in}	0	+	0	-	1	-
+	$\psi_{in}*$	1	-	0	+	0	+	0	ψ_{in}	0	-	1	-	0	+
+	$\psi_{in}*$	1	-	0	+	0	+	0	ψ_{in}	0	-	1	+	1	-
+	$\psi_{in}*$	1	-	0	+	0	+	0	$\psi_{in}*$	1	-	0	+	0	+
+	$\psi_{in}*$	1	-	0	+	0	+	0	$\psi_{in}*$	1	-	0	-	1	-
+	$\psi_{in}*$	1	-	0	+	0	+	0	$\psi_{in}*$	1	+	1	-	0	+
+	$\psi_{in}*$	1	-	0	+	0	+	0	$\psi_{in}*$	1	+	1	+	1	-
+	$\psi_{in}*$	1	-	0	+	0	+	1	ψ_{in}	0	+	0	+	0	+
+	$\psi_{in}*$	1	-	0	+	0	+	1	ψ_{in}	0	+	0	-	1	-
+	$\psi_{in}*$	1	-	0	+	0	+	1	ψ_{in}	0	-	1	-	0	+
+	$\psi_{in}*$	1	-	0	+	0	+	1	ψ_{in}	0	-	1	+	1	-
+	$\psi_{in}*$	1	-	0	+	0	+	1	$\psi_{in}*$	1	-	0	+	0	+
+	$\psi_{in}*$	1	-	0	+	0	+	1	$\psi_{in}*$	1	-	0	-	1	-
+	$\psi_{in}*$	1	-	0	+	0	+	1	$\psi_{in}*$	1	+	1	-	0	+
+	$\psi_{in}*$	1	-	0	+	0	+	1	$\psi_{in}*$	1	+	1	+	1	-
+	$\psi_{in}*$	1	-	0	-	1	-	0	ψ_{in}	0	+	0	+	0	+
+	$\psi_{in}*$	1	-	0	-	1	-	0	ψ_{in}	0	+	0	-	1	-
+	$\psi_{in}*$	1	-	0	-	1	-	0	ψ_{in}	0	-	1	-	0	+
+	$\psi_{in}*$	1	-	0	-	1	-	0	ψ_{in}	0	-	1	+	1	-
+	$\psi_{in}*$	1	-	0	-	1	-	0	$\psi_{in}*$	1	-	0	+	0	+
+	$\psi_{in}*$	1	-	0	-	1	-	0	$\psi_{in}*$	1	-	0	-	1	-
+	$\psi_{in}*$	1	-	0	-	1	-	0	$\psi_{in}*$	1	+	1	-	0	+
+	$\psi_{in}*$	1	-	0	-	1	-	0	$\psi_{in}*$	1	+	1	+	1	-
+	$\psi_{in}*$	1	-	0	-	1	-	1	ψ_{in}	0	+	0	+	0	+
+	$\psi_{in}*$	1	-	0	-	1	-	1	ψ_{in}	0	+	0	-	1	-
-	$\psi_{in}*$	1	-	0	-	1	-	1	ψ_{in}	0	-	1	-	0	+
-	$\psi_{in}*$	1	-	0	-	1	-	1	ψ_{in}	0	-	1	+	1	-
+	$\psi_{in}*$	1	-	0	-	1	-	1	$\psi_{in}*$	1	-	0	+	0	+
+	$\psi_{in}*$	1	-	0	-	1	-	1	$\psi_{in}*$	1	-	0	-	1	-
-	$\psi_{in}*$	1	-	0	-	1	-	1	$\psi_{in}*$	1	+	1	-	0	+
-	$\psi_{in}*$	1	-	0	-	1	-	1	$\psi_{in}*$	1	+	1	+	1	-
+	$\psi_{in}*$	1	+	1	-	0	+	0	ψ_{in}	0	+	0	+	0	+
+	$\psi_{in}*$	1	+	1	-	0	+	0	ψ_{in}	0	+	0	-	1	-
+	$\psi_{in}*$	1	+	1	-	0	+	0	ψ_{in}	0	-	1	-	0	+
+	$\psi_{in}*$	1	+	1	-	0	+	0	ψ_{in}	0	-	1	+	1	-
+	$\psi_{in}*$	1	+	1	-	0	+	0	$\psi_{in}*$	1	-	0	+	0	+
+	$\psi_{in}*$	1	+	1	-	0	+	0	$\psi_{in}*$	1	-	0	-	1	-
+	$\psi_{in}*$	1	+	1	-	0	+	0	$\psi_{in}*$	1	+	1	-	0	+
+	$\psi_{in}*$	1	+	1	-	0	+	0	$\psi_{in}*$	1	+	1	+	1	-
-	$\psi_{in}*$	1	+	1	-	0	+	1	ψ_{in}	0	+	0	+	0	+
-	$\psi_{in}*$	1	+	1	-	0	+	1	ψ_{in}	0	+	0	-	1	-
+	$\psi_{in}*$	1	+	1	-	0	+	1	ψ_{in}	0	-	1	-	0	+
+	$\psi_{in}*$	1	+	1	-	0	+	1	ψ_{in}	0	-	1	+	1	-
-	$\psi_{in}*$	1	+	1	-	0	+	1	$\psi_{in}*$	1	-	0	+	0	+
-	$\psi_{in}*$	1	+	1	-	0	+	1	$\psi_{in}*$	1	-	0	-	1	-
+	$\psi_{in}*$	1	+	1	-	0	+	1	$\psi_{in}*$	1	+	1	-	0	+
+	$\psi_{in}*$	1	+	1	-	0	+	1	$\psi_{in}*$	1	+	1	+	1	-
+	$\psi_{in}*$	1	+	1	+	1	-	0	ψ_{in}	0	+	0	+	0	+
+	$\psi_{in}*$	1	+	1	+	1	-	0	ψ_{in}	0	+	0	-	1	-
+	$\psi_{in}*$	1	+	1	+	1	-	0	ψ_{in}	0	-	1	-	0	+
+	$\psi_{in}*$	1	+	1	+	1	-	0	ψ_{in}	0	-	1	+	1	-
+	$\psi_{in}*$	1	+	1	+	1	-	0	$\psi_{in}*$	1	-	0	+	0	+
+	$\psi_{in}*$	1	+	1	+	1	-	0	$\psi_{in}*$	1	-	0	-	1	-
+	$\psi_{in}*$	1	+	1	+	1	-	0	$\psi_{in}*$	1	+	1	-	0	+
+	$\psi_{in}*$	1	+	1	+	1	-	0	$\psi_{in}*$	1	+	1	+	1	-
-	$\psi_{in}*$	1	+	1	+	1	-	1	ψ_{in}	0	+	0	+	0	+
-	$\psi_{in}*$	1	+	1	+	1	-	1	ψ_{in}	0	+	0	-	1	-
+	$\psi_{in}*$	1	+	1	+	1	-	1	ψ_{in}	0	-	1	-	0	+
+	$\psi_{in}*$	1	+	1	+	1	-	1	ψ_{in}	0	-	1	+	1	-
-	$\psi_{in}*$	1	+	1	+	1	-	1	$\psi_{in}*$	1	-	0	+	0	+
-	$\psi_{in}*$	1	+	1	+	1	-	1	$\psi_{in}*$	1	-	0	-	1	-
+	$\psi_{in}*$	1	+	1	+	1	-	1	$\psi_{in}*$	1	+	1	-	0	+
+	$\psi_{in}*$	1	+	1	+	1	-	1	$\psi_{in}*$	1	+	1	+	1	-